

Solving Inequalities

Main Ideas

- Solve inequalities with one operation.
- Solve multi-step inequalities.

New Vocabulary

set-builder notation

GET READY for the Lesson

Kuni is trying to decide between two rate plans offered by a wireless phone company.

	Plan 1	Plan 2	
Monthly Access Fee	\$35.00	\$55.00	0
Minutes Included	400	650	7
Additional Minutes	40¢	35¢	

To compare these two rate plans, we can use inequalities. The monthly access fee for Plan 1 is less than the fee for Plan 2, \$35 < \$55. However, the additional minutes fee for Plan 1 is greater than that of Plan 2, \$0.40 > \$0.35.

Solve Inequalities with One Operation For any two real numbers, *a* and *b*, exactly one of the following statements is true.

a < b a = b a > b

This is known as the **Trichotomy Property**.

Adding the same number to, or subtracting the same number from, each side of an inequality does not change the truth of the inequality.

KEY C	ONCEPT	Properties of Inequality
Additi	on Property of Inequality	
Words	For any real numbers, a, b, and c:	Example 3 < 5
	If $a > b$, then $a + c > b + c$.	3 + (-4) < 5 + (-4)
If $a < b$, then $a + c < b + c$.	-1 < 1	
Subtra	ction Property of Inequality	
Words	For any real numbers, <i>a</i> , <i>b</i> , and <i>c</i> :	Example 2 > -7
If $a > b$, then $a - c > b - c$. If $a < b$, then $a - c < b - c$.	If $a > b$, then $a - c > b - c$.	2 - 8 > -7 - 8
	-6 > -15	

These properties are also true for \leq , \geq , and \neq .

These properties can be used to solve inequalities. The solution sets of inequalities in one variable can then be graphed on number lines. Graph using a circle with an arrow to the left for < and an arrow to the right for >. Graph using a dot with an arrow to the left for \leq and an arrow to the right for \geq .



Multiplying or dividing each side of an inequality by a positive number does not change the truth of the inequality. However, multiplying or dividing each side of an inequality by a *negative* number requires that the order of the inequality be *reversed*. For example, to reverse \leq , replace it with \geq .

KEY C	ONCEPT		Properties of Inequality
Multiplication Property of Inequality			
Words	For any real n	umbers, <i>a</i> , <i>b</i> , and <i>c</i> , where	Examples
	if $a > b$, then $ac > bc$.	-2 < 3	
	c is positive:	if $a < b$, then $ac < bc$.	4(-2) < 4(3)
		if $a > b$, then $ac < bc$.	-8 < 12 5 > -1
c is negative:	if $a < b$, then $ac > bc$.	(-3)(5) < (-3)(21) -15 < 3	
Division Property of Inequality			
Words	s For any real numbers, <i>a</i> , <i>b</i> , and <i>c</i> , where		Examples
c is positive:	if $a > b$, then $\frac{a}{c} > \frac{b}{c}$.	-18 < -9	
	if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.	$\frac{-18}{3} < \frac{-9}{3}$	
			-6 < -3
		if $a > b$, then $\frac{a}{c} < \frac{b}{c}$.	12 > 8
	c is negative:	if $a < b$, then $\frac{a}{c} > \frac{b}{c}$.	$\frac{12}{-2} < \frac{8}{-2}$
			-6 < -4

These properties are also true for \leq , \geq , and \neq .

Reading Math

Set-Builder Notation

 $\{x \mid x > 9\}$ is read the set of all numbers x such that x is greater than 9.

The solution set of an inequality can be expressed by using **set-builder notation**. For example, the solution set in Example 1 can be expressed as $\{x | x > 9\}$.



Study Tip

Solutions to Inequalities

When solving an inequality,

- if you arrive at a false statement, such as 3 > 5, then the solution set for that inequality is the empty set, Ø.
- if you arrive at a true statement such as
 3 > -1, then the solution set for that inequality is the set of all real numbers.

Solve Multi-Step Inequalities Solving multi-step inequalities is similar to solving multi-step equations.





Extra Examples at algebra2.com

Real-World EXAMPLE Write

Write an Inequality

DELIVERIES Craig is delivering boxes of paper. Each box weighs 64 pounds, and Craig weighs 160 pounds. If the maximum capacity of the elevator is 2000 pounds, how many boxes can Craig safely take on each trip?

- **Explore** Let b = the number of boxes Craig can safely take on each trip. A maximum capacity of 2000 pounds means that the total weight must be less than or equal to 2000.
- **Plan** The total weight of the boxes is 64*b*. Craig's weight plus the total weight of the boxes must be less than or equal to 2000. Write an inequality.

Craig's weight 160	plus +	the weight of the boxes 64b	is less than or equal to \leq	<mark>2000</mark> . 2000
160 + 64	$b \le 2000$	Original inequ	ıality	
64	$b \le 1840$	Subtract 160 f	from each side.	
	$b \leq 28.75$	Divide each si	ide by 64.	
	Craig's weight 160 160 + 64 64	$\begin{array}{ll} \text{Craig's} \\ \text{weight} & \text{plus} \\ 160 & + \\ 160 + 64b \leq 2000 \\ 64b \leq 1840 \\ b \leq 28.75 \end{array}$	Craig'sthe weight plusof the boxes 160 + $64b$ $160 + 64b \le 2000$ Original inequ $64b \le 1840$ Subtract 160 for b ≤ 28.75 Divide each s	Craig'sthe weightis less than of the boxesweightplusof the boxesor equal to160+ $64b$ \leq 160 + $64b \leq 2000$ Original inequality $64b \leq 1840$ Subtract 160 from each side. $b \leq 28.75$ Divide each side by 64.

Check Since Craig cannot take a fraction of a box, he can take no more than 28 boxes per trip and still meet the safety requirements.

CHECK Your Progress

4. Sophia's goal is to score at least 200 points this basketball season. If she has already scored 122 points, how many points does Sophia have to score on average for the last 6 games to reach her goal?

Police Personal Tutor at algebra2.com

You can use a graphing calculator to solve inequalities.

GRAPHING CALCULATOR LAB

Solving Inequalities

is greater than or equal to

The inequality symbols in the TEST menu on the TI-83/84 Plus are called *relational operators*. They compare values and return 1 if the test is true or 0 if the test is false.

You can use these relational operators to solve an inequality in one variable.

LOGIC = 2:≠ 3:> 4:≥ 5:< 6:≤

THINK AND DISCUSS

- **1.** Clear the Y= list. Enter $11x + 3 \ge 2x 6$ as Y1. Put your calculator in DOT mode. Then, graph in the standard viewing window. Describe the graph.
- **2.** Using the TRACE function, investigate the graph. What values of *x* are on the graph? What values of *y* are on the graph?
- 3. Based on your investigation, what inequality is graphed?
- **4.** Solve $11x + 3 \ge 2x 6$ algebraically. How does your solution compare to the inequality you wrote in Exercise 3?

 Study Tip

 Math Symbols

 < is less than;</td>

 is fewer than

 > is greater than;

 is no more than;

 is less than or

 equal to

 ≥ is at least;

 is no less than;

CHECK Your Understanding

Examples 1–3 (pp. 34–35)	Solve each inequality. The 1. $a + 2 < 3.5$	Solve each inequality. Then graph the solution set on a number line. 1. $a + 2 < 3.5$ 2. $11 - c \le 8$		
	3. $5 \ge 3x$	4. −0.6 <i>p</i> < −9		
	5. 2 <i>w</i> + 19 < 5	6. $4y + 7 > 31$		
	7. $n \le \frac{n-4}{5}$	8. $\frac{3z+6}{11} < z$		
Example 4 (p. 36)	9. SCHOOL The final grad average test score and scores are out of 100 ar the student need on the	9. SCHOOL The final grade for a class is calculated by taking 75% of the average test score and adding 25% of the score on the final exam. If all scores are out of 100 and a student has a 76 test average, what score does the student need on the final exam to have a final grade of at least 80?		

Exercises

HOMEWORK HELP		
For Exercises	See Examples	
10, 11	1	
12–15	2	
16–26	3	
27–32	4	

Solve each inequality. Then graph the solution set on a number line.

10. $n + 4 \ge -7$	11. $b - 3 \le 15$	12. 5 <i>x</i> < 35
13. $\frac{d}{2} > -4$	14. $\frac{g}{-3} \ge -9$	15. $-8p \ge 24$
16. $13 - 4k \le 27$	17. $14 > 7y - 21$	18. −27 < 8 <i>m</i> + 5
19. $6b + 11 \ge 15$	20. $2(4t+9) \le 18$	21. $90 \ge 5(2r+6)$
22. $\frac{3t+6}{2} < 3t+6$	23. $\frac{k+7}{3} - 1 < 0$	24. $\frac{2n-6}{5} + 1 > 0$

- **25. PART-TIME JOB** David earns \$6.40 an hour working at Box Office Videos. Each week 25% of his total pay is deducted for taxes. If David wants his take-home pay to be at least \$120 a week, solve $6.4x 0.25(6.4x) \ge 120$ to determine how many hours he must work.
- **26. STATE FAIR** Admission to a state fair is \$12 per person. Bus parking costs \$20. Solve $12n + 20 \le 600$ to determine how many people can go to the fair if a group has \$600 and uses only one bus.

Define a variable and write an inequality for each problem. Then solve.

- **27.** The product of 12 and a number is greater than 36.
- **28.** Three less than twice a number is at most 5.
- **29.** The sum of a number and 8 is more than 2.
- **30.** The product of -4 and a number is at least 35.
- **31.** The difference of one half of a number and 7 is greater than or equal to 5.
- **32.** One more than the product of -3 and a number is less than 16.

Solve each inequality. Then graph the solution set on a number line.

33. $14 - 8n \le 0$ **34.** -4(5w - 8) < 33**35.** 0.02x + 5.58 < 0**36.** 1.5 - 0.25c < 6**37.** $6d + 3 \ge 5d - 2$ **38.** 9z + 2 > 4z + 15**39.** 2(g + 4) < 3g - 2(g - 5)**40.** $3(a + 4) - 2(3a + 4) \le 4a - 1$ **41.** $y < \frac{-y + 2}{9}$ **42.** $\frac{1 - 4p}{5} < 0.2$ **43.** $\frac{4x + 2}{6} < \frac{2x + 1}{3}$ **44.** $12(\frac{1}{4} - \frac{n}{3}) \le -6n$

CAR SALES For Exercises 45 and 46, use the following information.

Mrs. Lucas earns a salary of \$34,000 per year plus 1.5% commission on her sales. If the average price of a car she sells is \$30,500, about how many cars must she sell to make an annual income of at least \$50,000?

- 45. Write an inequality to describe this situation.
- **46.** Solve the inequality and interpret the solution.

Define a variable and write an inequality for each problem. Then solve.

- **47.** Twice the sum of a number and 5 is no more than 3 times that same number increased by 11.
- **48.** 9 less than a number is at most that same number divided by 2.
- **49. CHILD CARE** By Ohio law, when children are napping, the number of children per childcare staff member may be as many as twice the maximum listed at the right. Write and solve an inequality to determine how many staff members are required to be present in a room where 17 children are napping and the youngest child is 18 months old.

Maximum Number of Children Per Child Care Staff Member
At least one child care staff member caring for:
Every 5 infants less than 12 months old (or 2 for every 12)
Every 6 infants who are at least 12 months old, but less than 18 months old
Every 7 toddlers who are at least 18 months old, but less than 30 months old
Every 8 toddlers who are at least 30 months old, but less than 3 years old

Source: Ohio Department of Job and Family Services

TEST GRADES For Exercises 50 and 51, use the following information. EXTRA PRACI See pages 892, 926. Flavio's scores on the first four of five 100-point history tests were 85, 91, 89, and 94. Math Senine Self-Check Quiz at **50.** If a grade of at least 90 is an A, write an inequality to find the score Flavio algebra2.com must receive on the fifth test to have an A test average. **51.** Solve the inequality and interpret the solution. Graphing Use a graphing calculator to solve each inequality. Calculator **52.** -5x - 8 < 7**53.** $-4(6x-3) \le 60$ **54.** $3(x+3) \ge 2(x+4)$ H.O.T. Problems..... **55. OPEN ENDED** Write an inequality for which the solution set is the empty set. **56. REASONING** Explain why it is not necessary to state a division property for inequalities. **57. CHALLENGE** Which of the following properties hold for inequalities? Explain your reasoning or give a counterexample. **a.** Reflexive **b.** Symmetric **c.** Transitive **58.** CHALLENGE Write a multi-step inequality requiring multiplication or division, the solution set is graphed below. -5 -4 -3 -2 -1 0 1 2 3 5

59. *Writing in Math* Use the information about phone rate plans on page 33 to explain how inequalities can be used to compare phone plans. Include an explanation of how Kuni might determine when Plan 2 might be cheaper than Plan 1 if she typically uses more than 400 but less than 650 minutes.

STANDARDIZED TEST PRACTICE

- **60. ACT/SAT** If *a* < *b* and *c* < 0, which of the following are true?
 - $\mathbf{I.} \quad ac > bc$

II. a + c < b + c

III. a - c > b - c

- A I only
- **B** II only
- C III only
- **D** I and II only

61. REVIEW What is the complete solution to the equation |8 - 4x| = 40?
F x = 8; x = 12
G x = 8; x = -12
H x = -8; x = -12

J
$$x = -8; x = 12$$



Solve each equation. Check your solutions. (Lesson 1-4)

62. |x - 3| = 17

63. 8|4x - 3| = 64

65. E-COMMERCE On average, by how much did the amount spent on online purchases increase each year from 2000 to 2004? Define a variable, write an equation, and solve the problem. (Lesson 1-3)

Name the sets of numbers to which each number belongs. (Lesson 1-2)

66. 31 **67.**
$$-4.\overline{2}$$
 68. $\sqrt{7}$

69. BABY-SITTING Jenny baby-sat for $5\frac{1}{2}$ hours on Friday night and 8 hours on Saturday. She charges \$4.25 per hour. Use the Distributive Property to write two equivalent expressions that represent how much money Jenny earned. (Lesson 1-2)





GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation. Check your solutions. (Lesson 1-4)

70. $ x = 7$	71. $ x+5 = 18$	72. $ 5y - 8 = 12$
73. $14 = 2x - 36 $	74. $10 = 2 w + 6 $	75. $ x + 4 + 3 = 17$